On the Nucleon Stability and Σ_N Term Puzzles

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Abstract

A stable soliton configuration for a nucleon emerges when the nucleon stability and Σ_N term discrepancy problems are studied semi-quantitatively within a local theory developed recently. The approach developed here goes beyond the mean field or Hartree–Fock approximation by taking into account of the non-perturbative wave function renormalization in a way that is consistent with the chiral Ward–Takahashi identities of QCD. The stability condition for a nucleon and the recently extracted nucleon Σ_N term are used to estimate the radius R of the soliton. It is found that 0.67 fm $\leq R \leq 0.78$ fm. A new mechanism for the stability of a nucleon is proposed. The discrepancy between the nucleon Σ_N term extracted from pion–nucleon scattering data and the one from baryonic spectra is resolved by assuming the existence of a metastable virtual color superconducting phase for the strong interaction vacuum. Under such a scenario, the difference between the energy density of the chiral symmetry breaking phase and the metastable color superconducting phase is found to be 0.41 GeV/fm$^3 \leq \Delta\epsilon \leq 0.85$ GeV/fm^3.

1 Introduction

Two puzzles for a nucleon, which is believed to connect to its chiral properties, remain to be explained theoretically. The first one concerns with the mechanism for the stability of a nucleon; the second one is the nucleon Σ_N term discrepancy problem, which is made more severe by recent analysis of the pion–nuclon scattering data.

1) Despite the success of the bag and/or non-topological solitonic type of models for a nucleon in the case of a single nucleon, a serious potential problem for these models exists when one tries to describe many nucleon systems, like a nucleus, base on these models. On the one hand, there are empirical evidences which imply that a nucleon inside of a nucleus keeps most of its identity. On the other hand, the theoretical models allow the solution for a nucleon inside a nucleus to change significantly compared to the free space solution to favor a bag or a soliton of multi-quarks rather than the phenomenologically successful picture of a nucleus that can be described using the nucleon and meson degrees of freedom. Albeit arguments for a kind of equivalence between these two pictures were put forward in literature, the quantitative justification of this picture in realistic situations is still lacking. It implies that perhaps certain elements of an essential physics have not been understood so far. It is expected that a study of this possibility using the local theory recently developed could lead to a way out of the dilemma [1, 2, 3, 4].

2) Due to the approximate chiral symmetry in strong interaction, which is only explicitly broken by a tiny current mass for the light quarks, the low energy dynamics involving pion is very accurately constrained by the chiral symmetry of QCD. Indeed the Goldberger–Treiman relation, the Adler–Weisberger relation and other chiral relations are confirmed by data [5, 6] even with an extension of the PCAC relation [7, 8], which includes the possibility that there is a vector type virtual color superconducting phase. Such an expected success fails to reappear in the next leading order in the current quark mass. The resulting nucleon σ_N term, which is a measure of the scalar charge of the nucleon multiplied by the current quark mass, seems to be poorly connected to the on shell pion nucleon compton scattering data. The analysis of the most current data

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[9, 10] generates even greater values for Σ_N on the Cheng–Dashen point than the old one [13]–making the descrepancy larger ($\sim 100\%$ effect). The lattice QCD evaluation [11, 12] of the same quantity also produces a larger value. Under the conventional chiral symmetry breaking picture, this is indeed very puzzling: why the leading order relations are so accurately confirmed by data and at the same time the next leading order one fails 100%, given the fact that the expected error should be at most around $m_\pi/M_A \sim 10\%$ with m_π the mass of a pion and $M_A \sim 1$ GeV. Assuming a large strangeness content for a nucleon may not be consistent with other observables for the nucleon [13]. The way to resolve the nucleon Σ_N problem without certain new insight about the structure of the nucleon is a challenge if the chiral symmetry is regarded to be only slightly broken at low energy, as it manifests itself in the leading order and in many other observables.

The reason for the leading order results not to be affected by whether or not there is a metastable color superconducting phase [7, 8] is not hard to understand. The quantities that are sensitive to the possible metastable color superconducting phases are evaluated on the pion mass shell, resulting in a pion dominance. The quantity, namely g_A, which is not evaluated on the pion mass shell is however a quantity that is insensitive to the low energy chiral dynamics. In fact the scale that determines the change of g_A with the momentum transfer is controlled by a scale of order $M_A \sim 1$ GeV [14]. The nucleon Σ_N term on the other hand does not enjoy such an insensitivity.

It is therefore interesting to study whether or not these difficulties of the conventional picture for the nucleon could provide a useful source of information for a semi-quantitative discussion of various basic properties of a nucleon in the light of the local theory developed recently.

There are several new features in the local theory. First of all, it is based on an 8–component "real" representation for fermions, which is not equivalent to the currently adopted frame–work for relativistic processes at finite density [1, 2, 3, 4]. Some of the advantages of this representation are discussed in Ref. [15]. Second, the statistical gauge invariance, dark component for local observables and the statistical blocking effects can be studied in the new frame-work [1, 2, 3, 4]. It would be interesting to study what is its implication on a possible simultaneous solution of the aforementioned problems. It serves as a further consistency check of the mechanisms proposed here and in our earlier publications.

Qualitative discussion of such a solution to the nucleon stability problem is proposed recently [1, 2, 3, 4], in which it is argued that the statistical blocking effects of the strong interaction vacuum state can be the underlying physical mechanism for the stability of a nucleon inside of a nucleus and nuclear matter[1]. It remains to be investigated quantitatively as to whether or not such a mechanism is right in the sense that 1) it respects constraints of the chiral Ward–Takahashi identities and 2) it could reproduce at least two of the most important physical properties of the nucleon, namely, its size and its mass with reasonable model assumptions and parameters. This work aims at a semi-quantitative study of this question. A model picture for the nucleon has to be established and the physics and order of magnitude estimate of the possible quantitative properties of the model could then be derived.

The nucleon Σ_N term problem is also not a qualitative problem in the local theory if one assumes that there is at least one metastable color superconducting phase [7]. New developments are made both in the theoretical and phenomenological directions [16], which indicate that such a possibility can be oberved and further more is favored by the evidences considered. However these studies contains very little specific properties of the nucleon structure. It seems that a more quantitative study of the nucleon Σ_N problem is the natural next step.

The paper is organized in the following way. Section 2 contains an introduction of the chiral models for low and intermediate energy strong interaction, a discussion of the vacuum phenomenology in terms of wave function renormalized quasi-particles of the spontaneous chiral symmetry breaking, an introduction of a model for a nucleon and a determination of its size under such a scenario when the value of Σ_N is kept constant. A semi-quantitative mechanism for the stability of a nucleon inside of a nucleus and/or nuclear matter is suggested and discussed in Section 3. It is based upon the local theory in which the fermion fields are represented by an 8-component "real" spinor. The possible role played by the metastable color superconducting phases for the strong interaction vacuum state is discussed in section 4. The discrepancy of the nucleon Σ_N term extracted from different sources is resolved by assuming the existence of a virtual color superconducting phase for the strong interaction vacuum state, in which, a virtual color superconducting component for a nucleon lives. The difference in energy density between the true chiral symmetry breaking ground state and the virtual color superconducting phase is estimated based on available data. The section 5 contains further discussions. A summary is given in the last section.

2 The Global Σ_N Term and a Model for Nucleons

The standard model for strong interaction is QCD from which relevant non-perturbative information about the vacuum and hadron structure interested in this study is not easy to extract. Although lattice simulation could provide a first principle investigation of the problem, it is still not satisfactory. Full QCD is also too complicated for the present purpose. Simplified models constructed based on the slightly (explicit) broken chiral symmetry of the QCD Lagrangian in the fermionic sectors are frequently used, which are regarded as corresponding to the effective Lagrangian of QCD when the gluonic degrees of freedom are functionally integrated out. This effective Lagrangian is further simplified by assuming that only 4-point contact 4-point quark–quark interaction terms are dominant. The well known 't Hooft interaction due to instanton [17] is one of them. This kind of models are quite successful in describing a large collection of the hadronic phenomena (see, e.g., Refs. [18, 19, 20, 21, 22] and the references therein). It indicates that these models contain elements of the truth about the physics of strong interaction.

In the present section, attempts are made in the following to justify the picture that nucleons are made up of loosely bound constituent quarks in a chiral bag. The mass and wave function renormalization of these dressed quarks are generated by the spontaneous breaking of the chiral symmetry, which is governed by an 4–point quark–quark interaction. Albeit the long range confinement effects are not considered in these models, they are expected to be reasonable ones for the ground state chiral properties of hadrons, in which quarks are not separated far apart. The underlying QCD plays very little direct role as far as the chiral properties are concerned once these quantities are renormalized using relevant data from observations. Other quantities of interest here that detailed QCD dynamics may play more important role like the confinement, the wave function renormalization of the quark fields, the bag constant and the energy density of the possible virtual color superconducting phase are, instead of predicted by the models, fitted to experimental data. If possible, a comparison with available lattice QCD data is going to be carried out to test the consistency of the model assumptions.

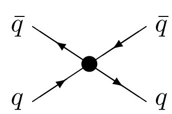
2.1 The strong interaction models

The 4–fermion interaction models for the light quark system involving up and down quarks have the generic form

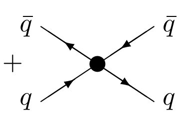
$$\mathcal{L} = \overline{\psi}\,(i\partial\!\!\!/ - m_0)\,\psi + \mathcal{L}_{int} \tag{1}$$

with the 4–fermion interaction terms classified into two categories in the quark–antiquark channel

$\mathcal{L}_{int} =$



$\Big\}$ *color singlet* $+$



$\Big\}$ *color octet* $+$ *Fierz term*

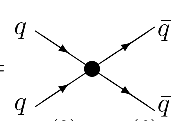
$$= \mathcal{L}^{(0)}_{int} + \mathcal{L}^{(8)}_{int}. \tag{2}$$

$\mathcal{L}^{(0)}_{int}$ generates quark–antiquark scattering in color singlet channel and $\mathcal{L}^{(8)}_{int}$ generates quark–antiquark scattering in color octet channel. For $\mathcal{L}^{(0)}_{int}$, the well known two flavor chiral symmetric Nambu Jona–Lasinio (NJL) interaction can be chosen, namely

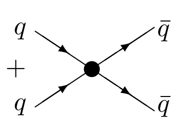
$$\mathcal{L}^{(0)}_{int} = G_0\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma^5\boldsymbol{\tau}\psi)^2\right]. \tag{3}$$

The color octet $\mathcal{L}^{(8)}$ is written in the quark–quark (antiquark–antiquark) channel form for our purposes, namely

$\mathcal{L}^{(8)}_{int} =$



$\Big\}$ *color triplet* $+$



$\Big\}$ *color sextet* $+ (q \leftrightarrow \overline{q})$

$$= \mathcal{L}^{(3)}_{int} + \mathcal{L}^{(6)}_{int}. \tag{4}$$

The color sextet term is repulsive in the one gluon exchange case. Due to the non-existence of colored baryon containing three quarks in nature, it is assumed to be generally true. So we restrict ourselves to the attractive color triplet two quark interaction terms. The attractive color triplet quark bilinear terms can be

classified according to their transformation properties under Lorentz and chiral $SU(2)_L \times SU(2)_R$ groups [23]. In general, if only terms without derivative in fermion fields are considered, $\mathcal{L}^{(3)}_{int}$ has the following form

$$\mathcal{L}^{(3)}_{int} = \frac{1}{2}\sum_r G_r \sum_{ab} C^r_{ab}(\overline{\psi}\Gamma^r_a\overline{\widetilde{\psi}})(\widetilde{\psi}\Gamma^r_b\psi), \tag{5}$$

with Γ^r_a, Γ^r_b matrices in Dirac, flavor and color spaces generating representation "r". The tilded fermion field operators $\widetilde{\psi}$ and $\widetilde{\overline{\psi}}$ in the 4-component representation are defined as

$$\widetilde{\psi} = \psi^T(-i\tau_2)C^{-1}, \tag{6}$$
$$\widetilde{\overline{\psi}} = Ci\tau_2\overline{\psi}^T. \tag{7}$$

In the 8-component representation for the fermion field in the local theory, these two identities are implemented as a pair of constraints.

Operator $\widetilde{\psi}\Gamma^r_b\psi$ belongs to an irreducible representation "r" of chiral, Lorentz and color groups and $\overline{\psi}\Gamma^r_a\widetilde{\overline{\psi}}$ belongs to the conjugate representation. Coefficients C^r_{ab} render the summation $\sum_{ab}\ldots$ invariant under Lorentz, chiral $SU(2)_L \times SU(2)_R$ and color $SU(3)_c$ groups. $\{G_r\}$ is a set of independent 4–fermion coupling constants characterizing the color triplet quark–quark interactions.

Since the QCD Lagrangian at low energy or long distance is presently unknown, the coupling constants G_r are not known. Albeit instanton induced 't Hooft interaction is frequently used in literature, which can reduce the independent coupling constant to only one, it will not be assumed here from the start. Rather, by studying the virtual phases of the strong interaction vacuum states using experimental observables, some of the important G_r, which determines the neighborhood of the strong interaction vacuum state, can at least be inferred. The question of whether or not instanton induced interaction dominates is left to be determined by observations.

For that purpose, two models that have different virtual superconducting phases for the vacuum state are studied. The half bosonized version of them with the quark field represented in an 8-component real representation [1, 23] are:

2.1.1 A model for scalar color superconductivity

$$\begin{aligned}\mathcal{L}_1 = {} & \frac{1}{2}\overline{\Psi}\left[i\partial\!\!\!/ - \sigma - i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma^5 O_3 - \gamma^5\mathcal{A}_c\chi^c O_{(+)} - \gamma^5\mathcal{A}^c\overline{\chi}_c O_{(-)}\right]\Psi \\ & -\frac{1}{4G_0}(\sigma^2+\boldsymbol{\pi}^2) + \frac{1}{2G_{3s}}\overline{\chi}_c\chi^c,\end{aligned} \tag{8}$$

where σ, $\boldsymbol{\pi}$, $\overline{\chi}_c$ and χ^c are auxiliary fields, $(\chi^c)^\dagger = -\overline{\chi}_c$, G_0 and G_{3s} are coupling constants of the model. The matrices O_i (i=1,2,3) are Pauli matrices acting on the upper and lower 4-components of Ψ and $\mathcal{A}_c$ (c=R,B,G) are a set of three 2×2 antisymmetric matrices acting on the color indices of Ψ.

This model Lagrangian [24, 1] has a color superconducting phase when the coupling constant G_{3s} is sufficiently large. When the phase in which the normal chiral symmetry breaking is the true ground state of the vacuum, the corresponding color superconducting phase is a metastable virtual phase [24, 1] in which the order parameter σ for the normal chiral symmetry breaking phase vanishes [16].

2.1.2 A model for vector color superconductivity

$$\begin{aligned}\mathcal{L}_2 = {} & \frac{1}{2}\overline{\Psi}\left[i\partial\!\!\!/ - \sigma - i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma^5 O_3 + O_{(+)}\left(\phi^c_\mu\gamma^\mu\gamma^5\mathcal{A}_c + \boldsymbol{\delta}^c_\mu\cdot\boldsymbol{\tau}\gamma^\mu\mathcal{A}_c\right)\right. \\ & \left.-O_{(-)}\left(\overline{\phi}_{\mu c}\gamma^\mu\gamma^5\mathcal{A}^c + \overline{\boldsymbol{\delta}}_{\mu c}\cdot\boldsymbol{\tau}\gamma^\mu\mathcal{A}^c\right)\right]\Psi - \frac{1}{4G_0}(\sigma^2+\boldsymbol{\pi}^2) \\ & -\frac{1}{2G_{3v}}(\overline{\phi}_{\mu c}\phi^{\mu c} + \overline{\boldsymbol{\delta}}_{\mu c}\cdot\boldsymbol{\delta}^{\mu c}),\end{aligned} \tag{9}$$

where $\overline{\phi}_{\mu c}$, ϕ^c_μ with $(\phi^\dagger_\mu)_c = -\overline{\phi}_{\mu c}$, $\overline{\boldsymbol{\delta}}_{\mu c}$, $\boldsymbol{\delta}^c_\mu$ with $(\boldsymbol{\delta}^\dagger_\mu)_c = -\overline{\boldsymbol{\delta}}_{\mu c}$ are auxiliary fields introduced.

This model Lagrangian [25, 23, 1] has a vector color superconducting phase induced by vector diquark condensation when the coupling constant G_{3v} is sufficiently large. The color superconducting phase also breaks the chiral symmetry. When the normal chiral symmetry breaking phase is the true ground state of the vacuum, the corresponding color superconducting phase is a metastable virtual phase [25, 23, 1] in which the order parameter σ for the normal chiral symmetry breaking phase vanishes [16].

2.1.3 The normal chiral symmetry breaking phase

The gap equation for the particles in the normal chiral symmetry breaking phase for both of these two models can be derived from a stability equation for the quark mass [23] in the Hartree–Fock approximation. The result is

$$\frac{\sigma^2}{\Lambda^2}\ln\left(1+\frac{\Lambda^2}{\sigma^2}\right) = \left(1-\frac{\pi}{12\alpha_0}\right) \tag{10}$$

with

$$\alpha_0 = \frac{\widetilde{G}_0\Lambda^2}{4\pi}, \tag{11}$$

where $\widetilde{G}_0$ is a linear combination of all the 4–point coupling constants of theory [23].

Since the vacuum $\sigma_{vac} \approx 350$ MeV, Eq. 10 implies $\alpha_0 \approx 0.38$ when Λ takes the value 0.9 GeV. The vacuum expectation value $< 0|\overline{\psi}\psi|0 >$ that can be extracted from Gell-Mann, Oakes and Renner [26] (GOR) relation

$$f_\pi^2 m_\pi^2 = -m_0 < 0|\overline{\psi}\psi|0 > \tag{12}$$

is related to σ_{vac} by

$$\sigma_{vac} = -2\widetilde{G}_0 < 0|\overline{\psi}\psi|0 >, \tag{13}$$

in the mean field or Hartree–Fock approximation. Here $m_0 = (m_u + m_d)/2$, m_u and m_d is the mass of the current up and down quark respectively.

For a realistic description of the physics of the strong interaction, the Hartree–Fock approximation may not be sufficient. One of the most important corrections is to sum over all the self energy diagrams for the auxiliary field $\sigma' = \sigma - \sigma_{vac}$, which is shown in Fig. 1. The result is a modification of the tree level

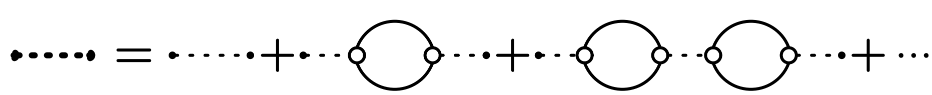


Figure 1: *The one loop correction to the σ' field propagator.*

propagator for σ', which is [23]

$$D_\sigma(p) = -2iG_0, \tag{14}$$

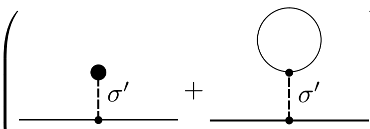
to

$$D_\sigma(p) = \frac{-2iG_0}{1-J_n(p)} \tag{15}$$

with $J_n \equiv J_n(0) > 0$. It causes the anti-screening of the scalar charge of a dressed quark.

The gap equation is however robust against this kind of corrections since the same $1/(1 - J_n)$ factor enters both of the tadpole terms and the Hartree part of the loop terms of the stability equation [23]. So the gap equation in the Hartree approximation with the dressed σ' propagator is of the following form

$$-i\delta\Sigma = \frac{1}{1 - J_n}\Bigg($$



$$\Bigg) = 0, \qquad (16)$$

where the larger black dot represents the tadpole due to a shift of the auxiliary field σ. The resummation over one-loop self-energy corrections to the σ' field propagator does not modify of the gap equation derived by using tree level propagator for the σ' field. The Fock terms for the stability equation can leads to a modification against the naive one loop equations due to the fact that the dressed propagator for σ' inside the loop depends on momentum and the Fock terms due to exchange of other auxiliary fields [1] does not renormalizes in the same way as σ'. This is especially true for the auxiliary fields corresponding to the $\mathcal{L}^{(8)}$ term. The difference should be ignored in the following discussion based on the large N_c argumentation which suppresses the Folk terms but should be studied in more detailed researches.

The Hartree–Fock relationship given in Eq. 13, on the other hand, has to be modified to

$$\sigma_{vac} = -\frac{2\widetilde{G}_0}{1 - J_n} < 0|\overline{\psi}\psi|0 > . \qquad (17)$$

This is because the effective 4-fermion interaction strength $\widetilde{G}_0^{eff} = \widetilde{G}_0/(1 - J_n)$ at the level of approximation considered. Since the resummation over the one-loop self-energy terms for the σ' field does not shift the pole position of the quark propagator, its effects on the quarks are represented by a wave function renormalization with $Z_\psi = 1 - J_n$, leading to a reduction of the vacuum expectation value of the $\overline{\psi}\psi$ operator against its mean field value.

2.2 The physical vacuum phenomenology and the global Σ_N term

2.2.1 The quasiparticle picture for the chiral condensate

The physics of the global observables are dominated by the quasiparticles of the true ground state of the vacuum [1, 2, 3, 4]. Its physical picture, which could be useful for the discussions in the following subsection, can be derived from the gap equation for the dynamical mass of the quarks. The generic form of the one loop gap equation is of the following form

$$\sigma_{vac} = i8n_f n_c \widetilde{G}_0 \int \frac{d^4p}{(2\pi)^4} \frac{\sigma_{vac}}{p^2 - \sigma_{vac}^2 + i\epsilon}, \qquad (18)$$

where $n_f = 2$ and $n_c = 3$ are the number of flavor and color respectively. Here, the current mass m_0 of the quarks is assumed to be zero for simplicity. If treated properly, the solution of this gap equation is beyond the Hartree–Fock approximation [23].

In order to project out the contributions of the quasiparticles of the true ground state of the vacuum, the 4–momentum integration can be carried out by doing the p^0 integration first [1, 3, 4]. The result is

$$\sigma_{vac} = 4n_f n_c \widetilde{G}_0 \int^{\Lambda_3} \frac{d^3p}{(2\pi)^3} \frac{\sigma_{vac}}{E_p} \qquad (19)$$

with $E_p = \sqrt{p^2 + \sigma^2}$ the energy of the quasiparticle and Λ_3 the formal cut off in the 3-momentum, which is not going to be used for the numerical evaluations. From Eq. 17, the scalar density of the vacuum state in

[1]The physical Goldstone pion contributions should be treated using a derivative coupling to the quarks to implement the chiral symmetry rather than using the pseudo-scalar coupling of the tree level $\boldsymbol{\pi}$ field [27]. This could leads to a significant reduction of the pion loop effects, e.g., at finite temperature [27].

the one loop approximation is

$$< 0|\overline{\psi}\psi|0 >= 2n_f n_c \int^{\Lambda_3} \frac{d^3p}{(2\pi)^3} \left[-(1-J_n)\frac{\sigma_{vac}}{E_p} \right] = 2n_f n_c \int^{\Lambda_3} \frac{d^3p}{(2\pi)^3} (1-J_n)\overline{u}(p)u(p) \tag{20}$$

with $u(p)$ a solution of the Dirac equation for a fermion with mass σ and energy $-E_p$. It can be seen that the non-perturbative[2] factor $Z_\psi = 1 - J_n$ serves as a wave function renormalization factor for the fermion field ψ.

Eq. 19 implies that the scalar charge density of the vacuum state is obtained by summing over the scalar charge

$$\lim_{q_\mu \to 0} j_s^{(-)}(p+q, p) = \overline{u}(p)u(p) \tag{21}$$

of contributing negative energy fermions in unit space volume renormalized by factor $1 - J_n$.

The physical value for $< 0|\overline{\psi}\psi|0 >$ can be computed from model independent GOR [26] relation, which is

$$< 0|\overline{\psi}\psi|0 >= -0.024 \ \text{GeV}^3 \tag{22}$$

when $m_0 = (m_u + m_d)/2 = 7$ MeV is assumed.

The value of σ_{vac} is generally taken to be around 0.35 GeV, which corresponds to the valence quark mass. Assuming $\Lambda = 0.9$ GeV, then Eq. 22 requires

$$J_n = 0.21 \tag{23}$$

which is a positive number as expected. This number is consistent with the lattice result $Z_\psi = 0.7 \sim 0.8$ of Ref. [28].

2.2.2 Chiral symmetry and pion decay constant

One of the model independent consequences of the chiral symmetry of the QCD Lagrangian is derived from the chiral Ward–Takahashi identity [8]

$$q^\mu \mathbf{\Gamma}^5_\mu = \frac{1}{2} m_0 D_\pi \gamma^5 \boldsymbol{\tau} - \frac{1}{4}\left\{\Sigma, \gamma^5 \boldsymbol{\tau}\right\} \tag{24}$$

with D_π a scalar function and $\mathbf{\Gamma}^5_\mu$ the proper axial-vector vertex for quarks that is dominated by the one pion pole at low q^2 and Σ their self-energy. $\boldsymbol{\tau}$ is the set of Pauli matrices in the isospin space. It leads to the GOR relation [7, 8] as the first order correction to the chiral symmetric results for the vacuum case, and, in certain approximate scheme like the one loop one, an evaluation of the decay constant for the chiral Goldstone boson [19, 23], namely,

$$f_\pi^2 = \frac{n_f n_c \sigma_{vac}^2}{8\pi^2} \left(\ln \frac{\sigma_{vac}^2 + \Lambda^2}{\sigma_{vac}^2} - \frac{\Lambda^2}{\sigma_{vac}^2 + \Lambda^2} \right). \tag{25}$$

It is one of the relationships used to fix the model parameters in the one loop approximation (see, e.g., [19]).

The picture developed here goes beyond the one loop approximation by formally summing over the self-energy terms for the auxiliary σ' fields, which results in a wave function renormalization factor $1 - J_n$ for the quasi-particles. Before the spontaneous chiral symmetry breaking taking place, the chiral Ward–Takahashi identity requires that there should also be a $1/(1 - J_n)$ renormalization factor for the axial-vector vertex as it is shown in the first graph of Fig. 2. After the chiral symmetry is spontaneously broken down, a massless Goldstone pion pole appears in the axial-vector vertex which is shown in the second graph of Fig. 2. It is reasonable to assume that the same renormalization for the pion–quark coupling vertex. The self-energy Σ that enters Eq. 24 is related to the quark mass σ_{vac} as

$$\Sigma \quad \to \quad \frac{\sigma_{vac}}{1 - J_n}. \tag{26}$$

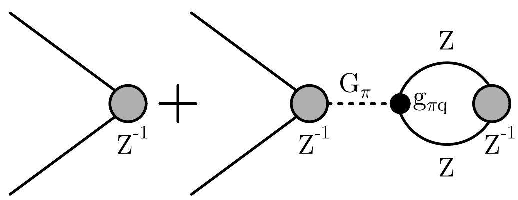


Figure 2: *The graphical representation of the full axial-vector vertex function for quarks. Here the grey blobs represent the corresponding the one pion irreducible proper vertices. The black dot represents the bare pion–quark coupling $g_{\pi q}$. Only the non-perturbative renormalization factors due to the self-energy for the σ' field, as indicated in the figure, are taken into account here.*

The various renormalization factors are displayed in Fig. 2. for the axial-vector vertex function with the pion pole separated out. The renormalized one loop result for the pion decay constant is then found to be

$$f_\pi^2 = \frac{n_f n_c \sigma_{vac}^2}{8\pi^2}(1-J_n)\left(\ln\frac{\sigma_{vac}^2+\Lambda^2}{\sigma_{vac}^2} - \frac{\Lambda^2}{\sigma_{vac}^2+\Lambda^2}\right). \tag{27}$$

The values $\Lambda = 0.9$ GeV, $f_\pi = 93$ MeV, $\sigma_{vac} \sim 350$ MeV and $J_n = 0.21$ adopted above are consistent with this equation and thus with the chiral Ward–Takahashi identity 24.

Thus, the approximation approach taken here, which improves the canonical quasi-particle picture by introducing a non-perturbative wave function renormalization factor $1 - J_n$, is quit successful in the vacuum sector.

2.2.3 Model independent information about the scalar charge (density) inside a nucleon

In the chiral quark model adopted here, the global nucleon Σ_N term, which can be extracted from the low energy pion–nucleon Compton scattering, can be evaluated in the following way

$$\Sigma_N(0) =< N| : \overline{\psi}\psi : |N >= m_0\left[3(1-J_n)j_s^{(+)} + \Delta v\delta < 0|\overline{\psi}\psi|0>\right]. \tag{28}$$

The scalar charge for the valence quarks $j_s^{(+)} \approx 1$, Δv is the volume taken by a nucleon and $\delta < 0|\overline{\psi}\psi|0 >$ is the change of the scalar charge density of the quarks on the negative energy states inside of the nucleon.

The global $\delta < 0|\overline{\psi}\psi|0 >$ term interested here is allowed to differ from zero. It is the sum of all the renormalized quark scalar charge density of all the contributing quarks orbits with a distortion of σ_{vac} inside of a nucleon minus those without a distortion of σ_{vac}. A non-vanishing value for this quantity thus implies a bag picture for a nucleon with the scalar field σ acting as a medium which is distorted to form the non-topological "bag".

Assuming that $m_0 = 7$ MeV, and that $\Sigma_N(0) \approx 55 \sim 75$ MeV after extracting from the Cheng–Dashen point [9, 10] to the $t = 0$ point, the vacuum polarization effects are

$$\Delta v\delta < 0|\overline{\psi}\psi|0 >= 5.5 \sim 8.4 \tag{29}$$

which is relatively large. As it is discussed in the following, it leads to a non-topological bag picture for a nucleon.

Before continuing, it should be pointed out that some of the studies of the nucleon Σ_N problem in the constituent quark model (see, e.g. Ref. [20] and many others) are based on the following equation

$$\Sigma_N(0) =< N| : \overline{\psi}\psi : |N >= m_0\frac{3j_s^{(+)}}{1-J_n}. \tag{30}$$

[2] The perturbative wave function renormalization around the σ_{vac} is absent since the radiative corrections to the self-energy of the quarks is self-consistently adjusted to zero in the auxiliary field approach [23] adopted here.

It can accommodate the old value $\Sigma_N(0) \sim 45$ MeV [20]. Such an equation is obtained by observing that $\Sigma_N(0)$ is a sum of the static scalar charge of quarks in an additive quark model. The effect of interaction can be obtained from the scalar vertex function by summing over ladder diagrams of the form given by Figs. 1 and 3, which gives us the factor $1/(1-J_n)$ in the above equation. The wave function renormalization of the two amputated quark legs in such an approach is not taken into account at the same level of approximation as the vertex function.

The inclusion of the effects of the wave function renormalization is required from unitarity point of view. The interaction of the of the self-energy graphs in Fig. 3 generates poles in the s-channel, which correspond to the physical mesons [29], these new poles should take some of the strength of the quasi-quark due to unitarity constraint. Without $1-J_n$ term, the so called "double or over counting" of the degrees of freedom is going to happen. Therefore Eq. 30 is not consistent since it is well known from the study of the

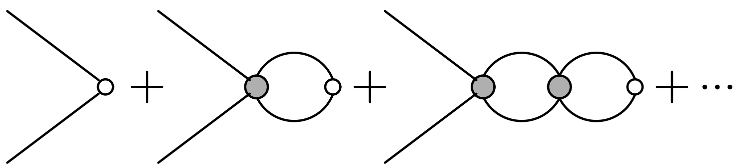


Figure 3: *The ladder diagrams for scalar charge vertex function. The 4–point vertex is shown in Fig. 4.*

gauge invariance in perturbative quantum electrodynamics that it is important to consider both the vertex function renormalization and the wave function renormalization in a given order in order to maintain the Ward–Takahashi identity. To sharpen the problem, it should be noticed that Eq. 20 should be

$$< 0|\overline{\psi}\psi|0 >= 2n_f n_c \int^{\Lambda_3} \frac{d^3p}{(2\pi)^3}\frac{\overline{u}(p)u(p)}{1-J_n} \tag{31}$$

in such an approximation. It would lead to a significant inconsistency between Eq. 22 and the phenomenological values $\sigma_{vac} = 0.35$ GeV, $\alpha_0 = 0.38$, $\Lambda = 0.9$ GeV, the value of the nucleon $\Sigma_N(0)$ value and the requirement that $J_n = 0.2 \sim 0.3$ from lattice simulation of QCD [28].

Under Eq. 28, it is inevitable to include a non-vanishing value for $\delta < 0|\overline{\psi}\psi|0 >$, as given by Eq. 29. It implies a non-topological bag picture for the nucleon.

The radius of the bag can be determined in the local theory [1, 2, 3, 4] by requiring that a nucleon is stable in a nucleus, which is revealed in experimental observations. In the aforementioned local theory, the energy of the lowest energy level in the bag must be at

$$E = \epsilon_0 - \mu = 0. \tag{32}$$

This is explained in the next section. Here μ is the time component of the statistical gauge field that is related to the fermion number density through [1, 2]

$$\rho = \frac{2}{\pi^2}\mu^3 \tag{33}$$

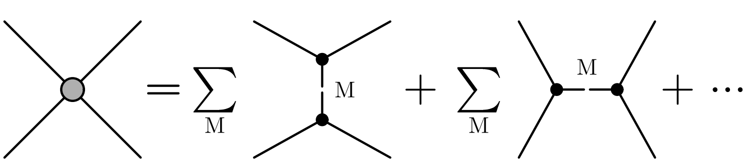


Figure 4: *The 4–point vertex in the scalar channel in Fig. 8. The summation is over all the auxiliary fields (M) introduced.*

when the value of the statistical blocking parameter ε_{vac} is much smaller than μ which is non-vanishing inside of a nucleon. Assuming that μ is constant inside the bag and zero outside, its value inside of the bag is related to the bag radius

$$\frac{2}{\pi^2}\Delta v\mu^3 = \frac{8}{3\pi}(R\mu)^3 = 3 \tag{34}$$

since the fermion number of a nucleon is 3. So

$$\mu = \frac{3}{2R}\left(\frac{\pi}{3}\right)^{1/3}. \tag{35}$$

The energy ϵ_0 can be estimated following the MIT bag model calculation

$$\epsilon_0 = \frac{c_0}{R} + \sigma_{in} \tag{36}$$

with $c_0 = 2.04$, σ_{in} the σ field inside of the bag, which is assumed zero in the authentic MIT bag model. Here, however,

$$\sigma_{in} = -\frac{2\widetilde{G}_0}{1-J_n}\left(< 0|\overline{\psi}\psi|0 > +\delta < 0|\overline{\psi}\psi|0 >\right). \tag{37}$$

Therefore $\epsilon_0 - \mu = 0$ implies

$$\left[c_0 - \frac{3}{2}\left(\frac{\pi}{3}\right)^{1/3}\right]\frac{1-J_n}{R} - \frac{6\alpha_0}{\Lambda^2}\frac{D}{R^3} = \frac{8\pi\alpha_0}{\Lambda^2} < 0|\overline{\psi}\psi|0 >, \tag{38}$$

where $D = \Delta v\delta < 0|\overline{\psi}\psi|0 >= 5.5 \sim 8.4$ (see Eq. 29). Substituting the values $\alpha_0 = 0.38$ and $\Lambda = 0.9$ GeV, we have

$$R = 0.67 \sim 0.78 \text{ fm} \tag{39}$$

for the core part of a nucleon. This is a quite reasonable range of numbers.

To get a rough picture for the distribution of σ field around a nucleon, let us estimate σ_{in} inside of a nucleon using the value obtained above

$$\sigma_{in} = -153 \sim -131 \text{ MeV} \tag{40}$$

Therefore we have a bag type of solution for a nucleon in this chiral model with its basic character determined by the measured global Σ_N term value. The value of the σ field outside of the bag is of order 350 MeV and the value for σ inside of the bag is of order $-153 \sim -131$ MeV.

The distribution of σ around a nucleon is schematically plotted in Fig. 5 in which the three black dots represent the energy level for the three valence quarks.

3 The Stability of a Nucleon

The stability of a nucleon inside a nucleus is an important enough problem to the considered, since, as far as it is known now, that the properties of a nucleus can be described well in terms of the nucleon degrees of freedom. On the other hand, the current theory tends to imply a change of the properties of a nucleon inside a nucleon. For example, the bag constant of a nucleon should be reduced inside of a heavy nucleus, which would give rise to an increase of the size of the nucleon. An overlapping configuration is also favored due the reduction in the surface area between different phases, which is not usually considered. There is little room for nucleons inside a heavy nucleus to increase their size before touch each other. There does not seems to be a reasonable scheme or mechanism to reconcile these two contradictory pictures at the present.

The stability problem of a nucleon inside of a nucleus is discussed qualitatively in Ref. [1] under the local theory for the relativistic finite density problems. The essential mechanism is derived from the statistical blocking effects in the normal chiral symmetry breaking phase of the strong interaction vacuum state that is revealed in the new theoretical frame-work.

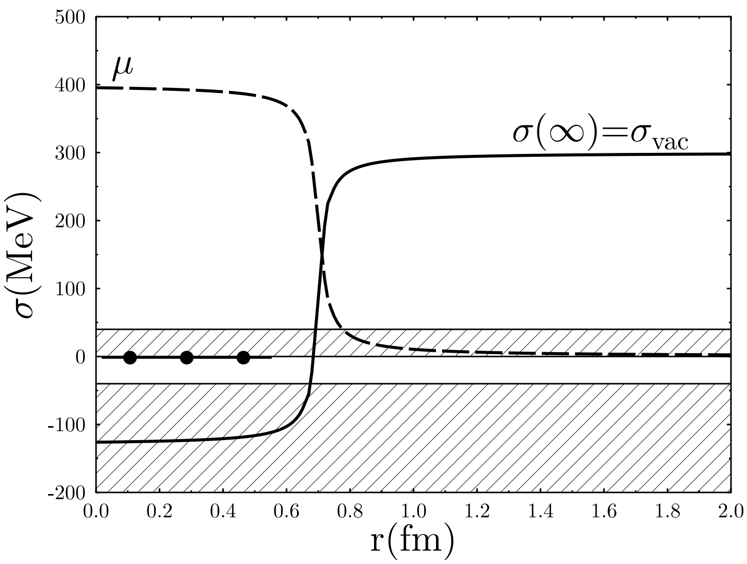


Figure 5: *A possible shape for the σ field around a nucleon that can reproduce the global $\Sigma_N(0)$ value. The shaded area contain the forbidden states due to the statistical blocking effects.*

Before getting into the quantitative discussions, an important qualitative property of the statistical blocking parameter ε^μ should be mentioned. It is related to the possibility of a distortion of the ε^μ around the nucleon, which seems to happen at a first look since the statistical gauge field μ^α has distinct different values inside and outside of a nucleon. Such a distortion does not happen for ε^μ. Because the statistical gauge field μ^α is coupled to a conserved current. A change of μ^α in a space-time region would cause a change of the baryon number density in the same region. Such a change can not be balanced locally since baryon number can not be created, it has to be transported inside or outside of the region from other regions far apart which cause corresponding changes of the baryon number in these other regions. In the end, the statistical gauge field μ^α correlates with each other at long distances[3]. The statistical blocking parameter does not couple to a conserved number; it can be seen as effectively couples to a current that is the difference of the fermion's baryon number and that of the anti-fermion's baryon number. This number can be created or destroyed locally (in space-time) in an extremely efficient way since a generation or annihilation of one fermion–anti-fermion pair inside of the vacuum would change two unit of the charge of this later effective current. This means that there is little correlation between ε^μ at different space-time points. Therefore, ε^μ can be viewed as corresponding to a very massive excitation of the system, which will not response to the presence of a nucleon once its vacuum expectation value is established.

3.1 The vacuum statistical blocking parameter

The absolute minima of the effective potential in the normal chiral symmetry breaking phase are located at non-vanishing values of ε [1, 2, 3, 4] and vanishing μ^α. The effective potential is of the following form

$$V_{\rm eff}(\sigma,\varepsilon) = i2n_f n_c \int_{\mathcal{C}} \frac{d^4p}{(2\pi)^4} \ln\left(1-\frac{\sigma^2}{p^2}\right) + \frac{1}{4\widetilde{G}_0}\sigma^2 + \frac{n_f n_c}{2\pi^2}\varepsilon^4, \tag{41}$$

where the dependency on the statistical gauge field μ^α is suppressed and the integration contour $\mathcal{C}$ lies beneath the real p^0 axis for $-\infty < p^0 < -\varepsilon$ and $0 < p^0 < \varepsilon$ and above the real p^0 axis when $-\varepsilon < p^0 < 0$ and $\varepsilon < p^0 < \infty$ [1]. The minima for $V_{\rm eff}(\sigma,\varepsilon)$ are not located at zero ε for any non-vanishing σ.

[3] Such a long distance correlation is destroyed by the statistical blocking effects in the normal chiral symmetry breaking phase or the α-phase of the strong interaction vacuum [1, 2, 3, 4].

For the case interested here, $\sigma = \sigma_{vac} = 350$ MeV. The value of the corresponding statistical blocking parameter ε_{vac} is found to be

$$\varepsilon_{vac} \approx 40 \text{ MeV} \tag{42}$$

after minimizing of the effective potential.

3.2 The energy of the valence quarks

In the 8–component "real" theory for fermions, there are two branches of single quark orbits with energy given by

$$E_{\pm} = \pm(\epsilon_0 - \mu) \tag{43}$$

that are relevant to the problem. Here ϵ_0 is given by Eq. 36 and μ is given by 35. For each specific value of R value, only one of these two states corresponds to particle excitation and the hole of the other state correspond to mirror antiparticles [15, 30]. The later excitation is absent in the conventional 4-component theory for fermions. The other states with

$$E'_{\pm} = \pm(\epsilon_0 + \mu) \tag{44}$$

lie in the continue spectra. They are not directly relevant to the discussion here.

The allowed state in the presence of the statistical blocking is shown in Fig. 6 for $\Sigma_N(0) = 62$ MeV. The spectrum of the particle excitation is draw with a solid line there. There are two discontinuities for the particle energy level for the valence quarks, with a gap of order of 80 MeV. The rest of the curves draw in dotted lines are for (mirror) anti-particles. It is likely that the crossing of E_+ and E_- at degenerate point of about $R \sim 0.72$ fm is separated by a gap in more quantitative studies. Such a small complication is not going to be discussed further here. The important feature here is the presence of a gap for the lowest energy of the quark orbits for bag radius $R \sim 0.70$ and $R \sim 0.74$ fm. These two gaps provide a stabilizing force for a nucleon.

3.3 The "quantization" of the size of a nucleon

The mass of a nucleon consists of two parts. The first one is the energy of the valence quarks discussed above. The second is the energy needed to establish the soliton configuration in which the value of σ is different from σ_{vac} and there is a finite statistical gauge field μ^0. Combining these contributions, the "classical value" of the mass is given by

$$M_N = 3E_0(R) + \frac{4\pi}{3}BR^3 + \frac{3}{2\pi^2}\Delta v\mu^4 - \frac{1.5}{R}, \tag{45}$$

where R is treated as classical quantity. Here $E_0(R)$ is the allowed energy for valence quarks, B is the bag constant similar to the MIT bag model and the third term is the corresponding term in the local theory [1]. The last term is a term from the free fermion orbits corresponding to the valence quarks that has to be subtracted according to the prescriptions of the local theory [1]. There should be a surface term that is proportional to R^2 in principle. I shall ignore such a term in the following discussions.

The value of B can in principle be obtained from V_{eff} too. But it will not be fixed this way here since the model for evaluating V_{eff} does not include many important elements such as the gluon degrees of freedom, which may be important especially when $\sigma \neq \sigma_{vac}$. It will be treated as free parameters here. If the value of $\Sigma_N(0)$ is chosen to be

$$\Sigma_N(0) = 62 \text{ MeV} \tag{46}$$

within the experimental range, then, it is found that $M_N \approx 1$ GeV if

$$B = (219 \text{ MeV})^4 = 2.3 \times 10^{-3} \text{ GeV}^4. \tag{47}$$

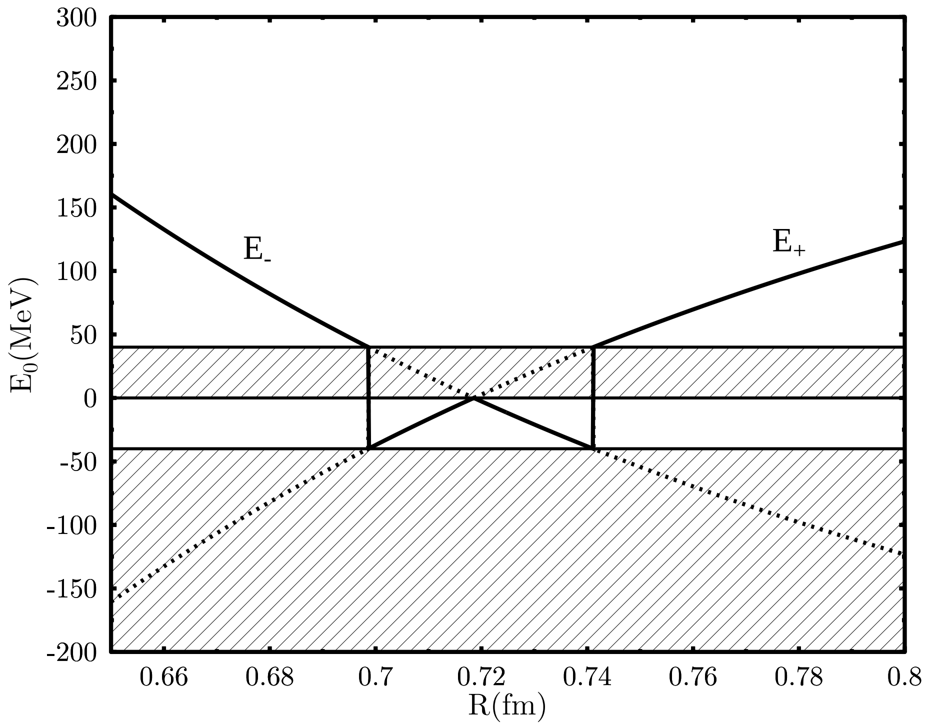


Figure 6: *The two branches of quark energy as functions of R in case of a constant $\Sigma_N(0) = 62$ MeV. When $R < 0.65$ fm, E_- is in the region of allowed state. The energy of the quark orbit is given by E_- in such a situation. As R is further increased, E_- gets into the blocked states and E_+ state becomes allowed. Such a switch of quark orbit release an energy of order* 80 *MeV for each quark. The role of E_+ and E_- is switched again at about $R \sim 0.72$ fm. It cause no energy in this case. The last switch of E_+ and E_- happens at $R \sim 0.72$ fm. An energy of order* 80 *MeV is needed to facilitate such a switch.*

The change of M_N with R is shown in Fig. 7. It can be seen that for the value of $\Sigma_N(0)$ chosen, the radius of a nucleon is around $0.67 \sim 0.78$ fm. Albeit there are two stable positions for M_N given in Eq. 45, a nucleon has a unique radius due to the fact that so far R is treated as a classical variable. Since R is a collective coordinate for a finite system of confined quarks (including the sea quarks), it has to be quantized, despite its fluctuation is expected to be much reduced [4], it is non-vanishing due to the finiteness of the system. If such a quantization of R is considered, the tunneling between these two stable position leads to a unique stable value for R, which is in the middle of the trapping well.

Also shown in Fig. 7 are M_N for different values of B. It can be seen that the location of the gap for M_N is not changed as B is varied despite the significant change of its value. This is an important property since one of the medium effects inside of a nucleus is represented by the reduction of B in the medium, which causes the increase of the size of the nucleon in the conventional picture. The statistical blocking effects in the local finite density theory prevent such a change from happening.

In the authentic MIT model, the dependence of M_N on the bag radius R is much flatter, which means that a nucleon is much softer than the one studied here. For example, the size of the nucleon changes rapidly from 0.75 fm to 1.1 fm as the bag constant $B^{1/4}$ is reduced from 219 MeV to 157 MeV, which is already too

[4] If a collective coordinate, like R considered here, connects to the common motion of N particles, the magnitude of its quantum spreading is reduced by a factor of $1/\sqrt{N}$ compared to the spreading of each of the single particles.

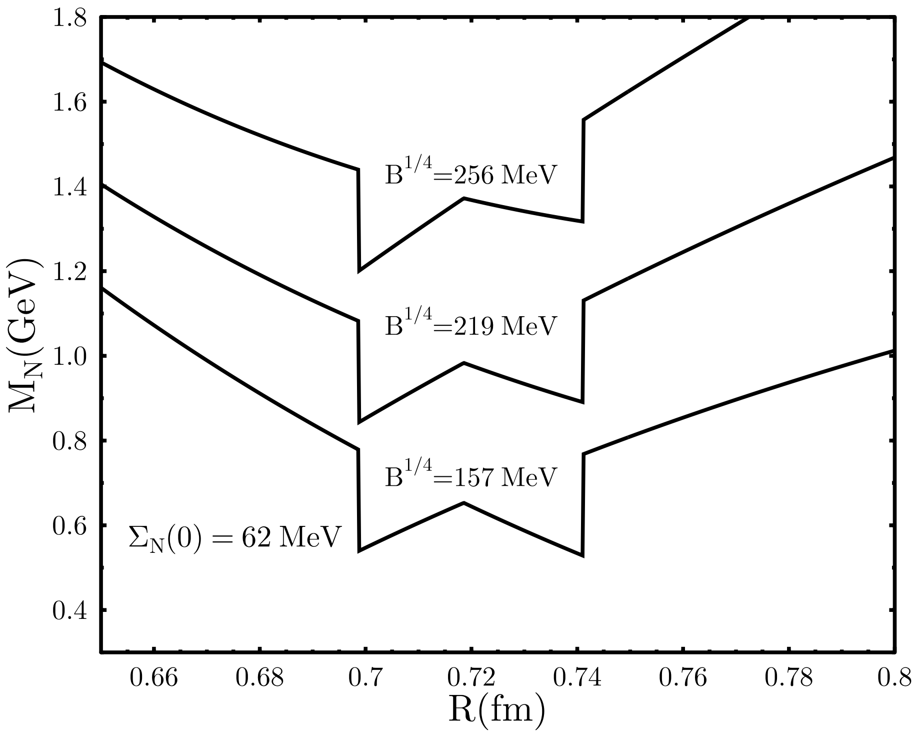


Figure 7: *The "classical" nucleon mass as a function of the soliton radius R for different values of the bag constant B. The value of $\Sigma_N(0)$ is assumed independent of R. The radius for the soliton is fixed against the change of B relative to the value that reproduces the nucleon mass.*

large.

Due to the presence of a gap of order 240 MeV for M_N, which is three times of the gap for a single valence quark, a nucleon should appear to be quite rigid against any form of external forces to change its radial size.

4 The Local σ_N Term and the Virtual Component of a Nucleon

After establishing a possible stable chiral model for a nucleon in a semi-quantitative way base on the phenomenological information about a nucleon, we now turn to the local properties of the nucleon concerning its possible color superconducting companion that lives in the possible virtual color superconducting phase of the strong interaction vacuum state. Such a possibility seems to be supported by various empirical evidences [31, 32, 33, 34]. The results of these works, however, depend only on general model independent properties of the virtual color superconducting phase, like the spontaneous partial breaking of the electromagnetic local gauge symmetry [31, 32, 33] and the spontaneous breaking of chiral symmetry [34]. Detailed structure of this possible superconducting component of a nucleon is not yet discussed.

A few properties of the color superconducting component of a nucleon that is likely to be true can nevertheless be listed. First, the radius of such a component should be comparable to the normal component discussed above. Second, there are also three valence quarks, which is the quasi-particles of the color

superconducting phase, in such a component. This is because a nucleon is color neutral. Third, the energy density of the observable color superconducting phase(s) should not be much larger than the true ground state of the vacuum. And fourth, the normal and the virtual component of a nucleon should appear in pair in low energy processes. Keeping this picture in mind, we shall turn to the question of the static scalar charge of a nucleon.

The nucleon Σ_N problem is different from the leading order relations because both side of the isoscalar part of the corresponding chiral Ward–Takahashi identity for the nucleon Σ_N problem are sensitive to the low energy chiral dynamics. The value for Σ_N on the Cheng–Dashen point extracted from pion–nucleon scattering data depends only on the pion nucleon coupling and, at low energy interested here, it is a global observable. The σ_N obtained from baryon spectrum contains the information of the possible metastable color superconducting phase [7] if it indeed exists. This is because it is a local operator in space-time defined on an equal time hypersurface within the spatial region occupied by the nucleon. Therefore, according to the local theory [1, 2, 3, 4] developed for such kind of situations, it contains the dark component that is not seen in the global low energy pion–nucleon scattering observables. It is therefore interesting to see to what extent one can learn, from the difference between the nucleon Σ_N term extracted from the pion–nucleon scattering data and the corresponding term extracted from the baryon mass spectra, about the structure of the companion virtual superconducting component of a nucleon that lives in the possible metastable color superconducting phase of the strong interaction vacuum state and also about the difference between the energy density of the true chiral symmetry breaking phase and the virtual color superconducting phase of the strong interaction vacuum.

4.1 The static scalar charge of a nucleon as a local observable

The scalar charge of the nucleon that is extracted from the baryon spectrum is the following expectation value of the normal ordered scalar operator

$$\sigma_N = m_0 \int d^3x < N| : \overline{\psi}\psi : (\boldsymbol{x}, t = 0)|N > . \tag{48}$$

The normal ordering is relative the vacuum state in which the chiral symmetry is spontaneously broken and it is measured at the equal-time hypersurface of the rest frame of the nucleon.

Although the detailed information about the right hand side (r.h.s) of Eq. 48 can not be easily extracted given the QCD dynamics, some general properties of it can nevertheless be given. First, the matrix element

$$< N| : \overline{\psi}\psi : (\boldsymbol{x}, t)|N >$$

is non-vanishing only around the nucleon in its rest frame due to the normal ordering. Therefore, the presence of a nucleon at the time $t = 0$ as specified in Eq. 48 can be viewed as a local perturbation (measurement) of the vacuum state with spatial dimension of order of the size of a nucleon and with a infinite resolution in time. According to the local theory developed in Refs. [1, 2, 3, 4], the quantity σ_N, which is a measure of certain component of the *energy density* inside a nucleon, contains a dark component.

The dark component can be classified into two kinds. The first kind of the dark component is from the deviation of the local observable from the one that contains only the contribution of the quasi–particles of the vacuum state. They are always present for local observables. For an semi-quantitative study given here, such a component is relatively hard to evaluate and is expected to be small. It is only included formally here. The second kind, which is perhaps more interesting, is due to the possibility that there is a close by metastable phase for the vacuum state. In this later case, the quasi-particles from the metastable phase also contribute to the local observables [1, 2, 3, 4] with a suppression factor $\mathcal{F}$ determined by the resolution of the observation $\Delta\omega$ and the difference $\Delta\epsilon$ between the energy density of the true phase and the virtual phase(s) of the vacuum state.

The suppression factor can be determined according to the prescription given in Refs. [1, 2, 3, 4]. First, the quantum correlation length for the observable interested is determined by the scale $1/M_A \sim 1$ GeV^{-1} ~ 0.2 fm, it is smaller than the size of the nucleon but is apparently larger than the inverse of the temporal resolution, which is zero. Therefore we have

$$\mathcal{F} = e^{-\Delta\epsilon\Delta v/M_A} \tag{49}$$

for all one particle irreducible graphs constructed from the quasiparticles of the virtual phase [1]. Here Δv is of the order of the volume taken by a single nucleon in space. Suppose that there is a virtual color superconducting phase, then Eq. 48 can be further specified as

$$\sigma_N = \sigma_n + \sigma_s \tag{50}$$

where σ_n is the contributions from the quasi-particles (together with their dark component of the first kind discussed above) of the ground state of the vacuum in which the chiral symmetry is spontaneously broken and σ_s is the contributions from the quasi-particles of the metastable virtual phase, which is assumed to be color superconducting here, including all the dark component of the first kind.

4.2 The local σ_N term

For the local observables, the two type of dark components discussed in section 4.1 has to be included. The dark components of the first kind are included formally by doing the loop 4-momentum integration in the Euclidean space which is cut off covariantly. The dark component of the second kind, namely, the one coming from the quasiparticles of the possible metastable phases of the vacuum, can be evaluated using standard field theoretical method.

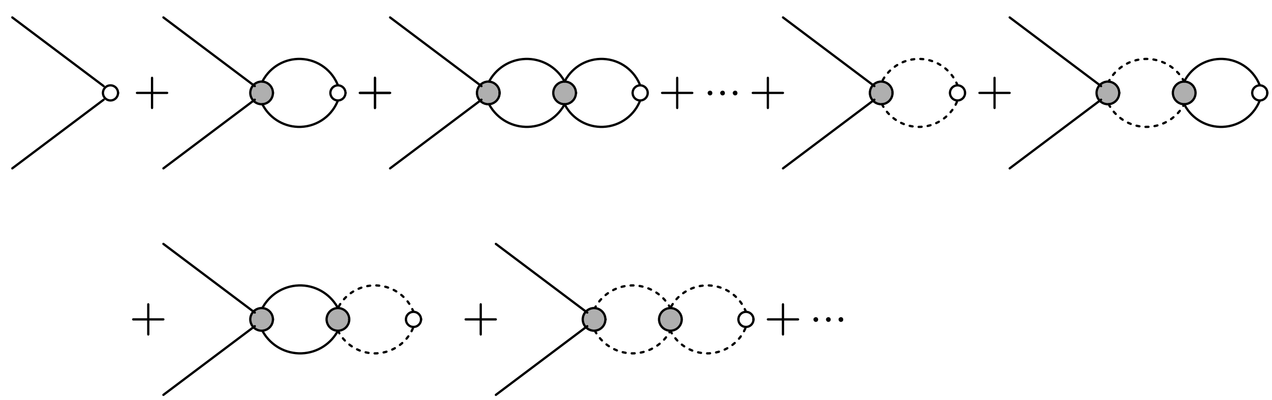


Figure 8: *The considered contribution to the local scalar vertex function of a quark in a 4–fermion interaction model. The lowest order graph in the upper row is the piece of the static charge of the quark in the mean field approximation that is usually included in the study of chiral symmetry breaking. The rest of the higher order graphs are the contributing one loop graphs in the conventional perturbation theories, which are represented by solid lines and the non-perturbative one loop contributions from the quasi-particles of the metastable phase whose propagator is represented by a dash line.*

The local scalar charge Eq. 48, can be written as

$$\sigma_N = \Sigma_N + \delta\sigma_N^{(val)} + \delta\sigma_N^{(sea)} \tag{51}$$

where Σ_N is the global value discussed above, $\delta\sigma_N^{(val)}$ is the contribution of the quasiparticles of the metastable phase to the valence quarks and $\delta\sigma_N^{(sea)}$ is the contribution of the quasiparticles of the metastable phase to the sea quarks.

The type of the corrections to the scalar charge of a quark are be diagrammatically expressed in Fig. 8. Since the valence quarks of the color superconducting component of a nucleon does not contain a scalar charge in the chiral limit. They do not contribute to the quantities studied here. Therefore only the Feynman graphs containing external quark lines living in the normal phase of the vacuum state are draw in Fig. 8.

After the summation of the series of the ladder diagrams and including the wave function renormalization

for the quark legs, the correction to the scalar charge of a quark is found to be

$$\delta j_s = -J_s \overline{u}(p) u(p) \tag{52}$$

with the one loop diagram contribution denoted by J_s. The value of J_s depends on the nature of the color superconducting phase.

4.2.1 The possible virtual scalar color superconducting phase

The propagators for the quarks in the scalar color superconducting phase can be found after fixing the complex phase and the direction in the color space of χ^c. If χ^c is chosen to be along, say, the "red" direction and to be real, then [24, 1]

$$S_{F1}(p) = \frac{i}{\not{p} - \gamma^5 \mathcal{A} \chi O_1} \tag{53}$$

for "blue" and "green" quarks and

$$S_{F2}(p) = \frac{i}{\not{p}} \tag{54}$$

for "red" quark.

The bubble diagram in Fig. 8 is found to be

$$\begin{aligned} J_s &= -i\widetilde{G}_0 \mathcal{F} \mathrm{Tr} \int \frac{d^4 p}{(2\pi)^4} \left[S_{F1}(p) S_{F1}(p) + S_{F2}(p) S_{F2}(p) \right] \\ &= 16 \widetilde{G}_0 \mathcal{F} \int \frac{d^4 p_E}{(2\pi)^4} \left(\frac{2}{p_E^2 + \chi^2} + \frac{1}{p_E^2} \right) \end{aligned} \tag{55}$$

with p_E^μ the Euclidean 4-momentum. The result is

$$J_s = \frac{12\alpha_0}{\pi} \mathcal{F} \left[1 - \frac{2}{3} \frac{\chi^2}{\Lambda^2} \ln \left(1 + \frac{\Lambda^2}{\chi^2} \right) \right] = \mathcal{F} \frac{1 - \frac{2}{3} \frac{\chi^2}{\Lambda^2} \ln \left(1 + \frac{\Lambda^2}{\chi^2} \right)}{1 - \frac{\sigma^2}{\Lambda^2} \ln \left(1 + \frac{\Lambda^2}{\sigma^2} \right)}. \tag{56}$$

Here, use has been made of the gap equation for σ, namely Eq. 10. Since $\sigma \approx 350$ MeV and assuming $\chi \sim 200$ MeV, then $J_s = 1.3 \times e^{-\Delta\epsilon \Delta v / M_A}$.

The contribution of the quasiparticles of the virtual phase to the value of the local $\sigma_N(0)$ can be obtained from Eq. 28 through a replacement of the global wave function renormalization factor $1 - J_n$ by local one, namely $1 - J_n \to 1 - J_n - J_s$. Since the wave function renormalization factor for every contributing quark orbits, including the sea quarks, is modified in such a way, we can write

$$\sigma_N(0) = \Sigma_N(0) - \frac{J_s}{1 - J_n} \Sigma_N(0). \tag{57}$$

Therefore we have

$$J_s = \frac{(1 - J_n) \Delta \Sigma_N}{\Sigma_N(0)}. \tag{58}$$

For the value of χ taken, we have

$$\Delta\epsilon = \frac{3}{4\pi} \frac{M_A}{R^3} \ln \frac{1.3 \times \Sigma_N(0)}{(1 - J_n) \Delta \Sigma_N(0)}. \tag{59}$$

Using the estimates $\Sigma_N(2\mu^2) - \Sigma_N(0) = 10 \sim 15$ MeV from the chiral perturbation theory [35] and dispersion relation calculation [36], $\Sigma_N(0) = 55 \sim 75$ MeV. If no strangeness is assumed for a nucleon and $M_A = \Lambda = 0.9$ GeV, then

$$\Delta\epsilon = 789 \sim 409 \ \ \mathrm{MeV/fm^3}. \tag{60}$$

This is a reasonable number.

4.2.2 The possible virtual vector color superconducting phase

The propagator represented by the dashed lines in Fig. 8 for the vector color superconducting phase are also found after fixing the color and complex phase for the order parameter ϕ^c_μ. One of the choices [23] is that for "blue" and "green" quarks

$$S_{F1}(p) = \left(1 - iO_2 \frac{\not p}{p^2} \not\phi \gamma^5 \mathcal{A}\right) F(p), \tag{61}$$

$$F(p) = i\frac{(p^2 - \phi^2)\not p - 2p \cdot \phi \not\phi}{(p^2 - \phi^2)^2 - 4(p \cdot \phi)^2} \tag{62}$$

and

$$S_{F2}(p) = \frac{i}{\not p} \tag{63}$$

for "red" quark.

The bubble diagram in Fig. 8 is found to be

$$\begin{aligned} J_s &= -i\widetilde{G}_0 \mathrm{Tr} \int \frac{d^4p}{(2\pi)^4} \left[S_{F1}(p)S_{F1}(p) + S_{F2}(p)S_{F2}(p)\right] \\ &= 16\widetilde{G}_0 \int \frac{d^4p_E}{(2\pi)^4} \left(\frac{2(p_E^2 + \phi^2)}{(p_E^2 + \phi^2) - 4(p_E \cdot \phi)^2} + \frac{1}{p_E^2}\right) \end{aligned} \tag{64}$$

with p^μ_E the Euclidean 4-momentum. The result is quite simple

$$J_s = \frac{12\alpha_0}{\pi} \mathcal{F}\left(1 - \frac{1}{3}\frac{\phi^2}{\Lambda^2}\right) = \mathcal{F} \frac{1 - \frac{1}{3}\frac{\phi^2}{\Lambda^2}}{1 - \frac{\sigma^2}{\Lambda^2} \ln\left(1 + \frac{\Lambda^2}{\sigma^2}\right)}. \tag{65}$$

For typical values of $\sigma_{vac} = 350$ MeV and $\sqrt{\phi^2} = 200$ MeV, $J_s = 1.4 \times e^{-\Delta\epsilon\Delta v/M_A}$ and Eq. 59 is replaced by

$$\Delta\epsilon = \frac{3}{4\pi}\frac{M_A}{R^3} \ln \frac{1.4 \times \Sigma_N(0)}{(1 - J_n)\Delta\Sigma_N(0)}, \tag{66}$$

which is not much different from Eq. 59. The values for $\Delta\epsilon$ is of the same order as Eq. 60, namely

$$\Delta\epsilon = 853 \sim 450 \ \ \mathrm{MeV/fm^3}. \tag{67}$$

This is a reasonable number.

This range of $\Delta\epsilon$ is quite stable against change of the order parameters for the color superconducting phase. For example, even in the limit $\sqrt{\phi^2} \to 0$,

$$\Delta\epsilon = 865 \sim 457 \ \ \mathrm{MeV/fm^3}, \tag{68}$$

which changes very little. The same is true for the scalar color superconducting phase.

5 Discussions

5.1 The inputs and outputs

The input parameters are listed in Table 1. The quantitative outputs of our investigation based on the local

Table 1: *The input parameters and equations. The value of J_n is consistent with the recent lattice evaluation. The value of c_0 is a prediction of the MIT bag model. It is treated as an input to reflect our ignorance of the shape of $\sigma(r)$.*

α_0	Λ	J_n	χ or $\sqrt{\phi^2}$	B	c_0
0.38	0.9 GeV	0.21	0.2 GeV	2.3×10^{-3} GeV^4	2.04

Table 2: *The physical/empirical values generated by the model in the local theory. These values are not fitted. They are derived values according to the local theory. The known quantities agree well with the popular ones in literature. The new parameters of the local theory, namely μ, $\Delta\epsilon$, ε_{vac} and the stability gap, are predicted in this work. All these quantities are consistent with the chiral symmetry encoded in the Ward–Takahashi (WT) identities. The number in bracket is the observed one. M_N^* correspond to the case of $\Sigma_N(0) = 62$ MeV. The unit for various quantities here is defined the text.*

Physical	f_π	M_N^*	$\Sigma_N(0)$	$\sigma_N(0)$	$< 0\vert\overline{\psi}\psi\vert 0 >$
Quantities	92 (93)	1.0 (0.938)	$55 \sim 75$ ($55 \sim 75$)	35 (35)	-0.024 (-0.024)
Physical	σ_{vac}	R	σ_{in}	μ	$\delta < 0\vert\overline{\psi}\psi\vert 0 >$
Quantities	350	$0.67 \sim 0.78$	$-0.15 \sim -0.13$	$0.45 \sim 0.38$	$0.034 \sim 0.032$
Physical	$\Delta\epsilon_s$ (Scalar)	$\Delta\epsilon_v$ (Vector)	ε_{vac}	Stability gap	
Quantities	$0.79 \sim 0.41$	$0.85 \sim 0.45$	40	~ 240	
Chiral Symmetry	Eq. 12	Eq. 27	Σ_N puzzle	Nucleon stability	
& Others	Satisfied	Satisfied	Solved	Ensured	

theory are given in Table 2. It can be seen that using 6 input parameters/equations, more than double physical quantities/equations can be obtained/satisfied. The gain is obvious. In addition, a new mechanism for the stability of a nucleon inside a nucleus or nuclear matter is found and semi-quantitative pictures for the nucleon and the vacuum state of the strong interaction is also established.

The presence of at least one metastable virtual color superconducting phase for the strong interaction vacuum is assumed instead of derived in this paper. Unlike in other work, like Refs. [33, 31, 34], the existence of the metastable color superconducting phase for the strong interaction vacuum state is a sufficient condition for the solution of the nucleon Σ_N term puzzle. It is not a necessary one despite the fact that such an assumption is rather natural for the solution of the puzzle.

5.2 The nucleon as a non-topological soliton

It is demonstrated that when the quark wave function renormalization is taken into account, the experimentally observed global nucleon Σ_N term implies, in general, that a modification of the quasi–particle picture for the hadrons seems to be inevitable. This kind of picture for a nucleon is somewhat different from the authentic constituent quark model but are in qualitative agreement with the bag type of models

[37, 38, 39, 40] for a nucleon introduced on more phenomenological basis where the nature of the scalar σ field, which is claimed to be related to gluon contributions, is actually not clear. Here the scalar field is the order parameter for the chiral symmetry, which contains contributions from gluon as well as quarks. The sizes of the non-topological soliton are found to be around 0.7 fm.

Given the fact that σ_{vac}, the bag constant B and the renormalization factor J_n are related to the complicated underlying QCD dynamics, they are not computed using the 4-fermion interaction models adopted here, which serve the purpose of providing the general structure of the solution to the problems and of discussing the chiral symmetry issues. Rather, these parameters are determined by fitting empirical data. This is done despite the fact that model computations may not leads to a set of values significantly deviate from the fitted ones, such a success may not be regarded as an indication that QCD formulated in the local theory can be replaced by the models but only that these models are effective ones.

5.3 The solution of the two puzzles

After the basic picture of a nucleon is established, the role played by the statistical blocking effects of the chiral symmetry breaking phase in maintaining the stability of a nucleon inside a large nucleus and in a nuclear matter is discussed. It is shown that two of the essential elements of the local theory, namely, the statistical blocking effects and the inequivalent 8-component "real" representation for the fermion field are both needed for the mechanism. The bag constant B is determined in the frame-work of the local theory by producing the right mass for the nucleon.

The discrepancy between the nucleon Σ_N term from low energy pion–nucleon scattering data and the one extracted from baryon spectra is resolved by realizing that the former one is a global observable while the later one is a local one, which according to the local theory, contains a dark component. The dark component is further attributed to a possible virtual color superconducting phase that seems to manifest in the high energy electromagnetic interactions involving a nucleon [31, 32, 33] and some other ones [16]. The difference in energy density of the metastable phase and the stable phase of the strong interaction vacuum state is estimated.

Thus, the new features of the local theory [1, 2, 3, 4] and the assumption that there is a virtual color superconducting phase for the strong interaction vacuum state provides a set of conditions for a simultaneous solution of the two puzzles.

5.4 The diquark model for a nucleon?

The diquark models for a nucleon has a long history. It has certain advantages on the phenomenological ground. There are theoretical difficulties for these models. It is criticized in a similar, albeit more detailed in certain aspects, approach to the quark–quark interaction, which does not consider the required wave function renormalization [29]. We belief that their argument is sound at least for this specific problem.

But under the current scenario, the result of Ref. [29] is not the end of the diquark model for a nucleon since its argument applies only to the normal component of the nucleon, which lives in the true ground state of the vacuum in which the chiral symmetry is spontaneously broken. In the case of the possible virtual color superconducting component for a nucleon, which lives in the possible virtual color superconducting phase of the vacuum, an entirely different set of channels, like in the χ or ϕ^μ, $\boldsymbol{\delta}^\mu$ etc. fields must be iterated. It is not known at the present whether or not a quark–quark clustering for the "constituent" quarks in the color superconducting component of a nucleon is favored or not. It is one of the questions to be understood in the future.

5.5 Speculations

Given these figures, it is tempting to speculate that perhaps the Ropper resonance at 1.44 GeV is an excited N^* resonance produced in such a way that its normal component is striped away by some means so that it is dominated by the color superconducting companion of a nucleon? It would be very interesting to understand the structure of the color superconducting virtual companion of the nucleon and more importantly the virtual color superconducting phase of the strong interaction vacuum state if they are indeed there.

6 Summary

A mechanism for the stability of a nucleon and a resolution of the nucleon Σ_N term problem are studied based on an implementation of the chiral Ward–Takahashi identity of QCD at a level that goes beyond the mean field approximation by formally summing over ladder diagrams for σ' field. The local theory for the problem [1, 2, 3, 4] is adopted as the theoretical frame-work for a coherent discussion and solution of these two seemingly unrelated puzzles.

In the future, self-consistent numerical study of the non-topological soliton model proposed here is an interesting topic to be studied [41]. Theoretical and experimental means should be searched and/or developed to study the structure of the possible superconducting component of the nucleon in more details. It is interesting not only to the nucleon structure itself, but also because it carries information about the possible virtual color superconducting phase of the vacuum state, which is one of the most important objects to be understood in the context of contemporary discoveries in cosmology.

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